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STRUCTURAL INELASTICITY XV

Limit Analysis and Coulomb Friction

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LIMIT ANALYSIS AND COULOMB FRICTION[1]

by

Philip G. Hodge, Jr.[2]

ABSTRACT

The theorems of limit analysis are not valid if part of the boundary is pressed against a rigid surface and constrained by coulomb friction. In reference to a problem of inverted extrusion, we show how conventional limit analysis can be extended to obtain upper and lower bounds on the force necessary for extrusion.

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LIMIT ANALYSIS AND COULOMB FRICTION

When part of a rigid/perfectly-plastic body is pressed against a surface and constrained by coulomb friction, then the normal velocity component will be zero. The tangential component will also be zero unless the shear traction stress is equal to μN where μ is the coefficient of friction and N is the compressive normal traction or is equal to the shear yield stress k. Thus

$$0 \leq T \leq \min(k, \mu N)$$

$$V_t \geq 0 \qquad (1)$$

$$\text{IF } V_t > 0 \text{ then } T = \min(k, \mu N)$$

where T and V_t are positive in opposing directions. Equations (1) do not explicity prescribe either stress or velocity, and hence the conventional theorems of limit analysis are not generally valid.

As an example of a case where this phenomenon occurs, we consider inverted extrusion through a tapered die with coulomb friction, Fig. 1. The yield-point load F is that value of the applied force which will just cause the billet to move through the die at an arbitrarily slow speed. We consider the case of plane strain; a similar result obviously applies to rotational symmetry.

We shall assume that at yield-point motion takes place all along the die face AB. Then the kinematic and static boundary conditions are, respectively

On ACC´A´: $V_x = V_1$, $V_y = 0$

On AB and A´B´: $V_n = 0$, $V_t > 0 \qquad (2)$

On BDD´B´: $V_x = V_2 = V_1 t_1/t_2$, $V_y = 0$

and

On AB and A´B´:

$$T = (1/2)(\sigma_y - \sigma_x)\sin 2\alpha - \tau_{xy}\cos 2\alpha \qquad (3a)$$

$$N = -(1/2)(\sigma_y + \sigma_x) - (1/2)(\sigma_y - \sigma_x)\cos 2\alpha - \tau_{xy}\sin 2\alpha \qquad (3b)$$

$$T = \min(k, \mu N) \qquad N \geq 0 \qquad (3c)$$

Over-all horizontal equilibrium of the billet shows that

$$t_1 F = \int_{S_2}^{S_1} (T\cos\alpha + N\sin\alpha)ds \qquad (4)$$

Since it follows from Eq. (3c) that $N \geq T/\mu$, Eq. (4) leads to the inequality

$$\int_{S_2}^{S_1} T ds \leq t_1 F(\cos\alpha + \sin\alpha/\mu)^{-1} \qquad (5)$$

We define a statically admissible stress field as stresses σ_{ij}° which satisfy internal equilibrium, the yield condition and boundary conditions (3), together with a load F° defined by (4). The principle of virtual work then shows that

$$\frac{1}{2}\int_A \sigma_{ij}^{\circ}\,\dot{\varepsilon}_{ij} = t_1 V_1 F^{\circ} - \int_{S_2}^{S_1} T^{\circ} V_t ds \qquad (6a)$$

$$\frac{1}{2}\int_A \sigma_{ij}\dot{\varepsilon}_{ij} = t_1 V_1 F - \int_{S_2}^{S_1} T V_t ds \qquad (6b)$$

Subtracting (6a) from (6b) we obtain

$$t_1 V_1 (F - F^{\circ}) - \int_{S_2}^{S_1} (T - T^{\circ}) V_t ds = \frac{1}{2}\int (\sigma_{ij} - \sigma_{ij}^{\circ})\dot{\varepsilon}_{ij} \geq 0 \qquad (7)$$

where the last step follows from Drucker's postulate [1] for a perfectly-plastic material.

Since T°, T, and V_t are each non-negative at each point, Eq. (7) implies the weaker restriction

$$t_1V_1F - 0 \geq t_1V_1F° - (V_t)_{max} \int_{S_2}^{S_1} T°ds \qquad (8)$$

Finally, we make the physically reasonable assumption that since the frictional force is restraining the material at the boundary, the horizontal component of the boundary velocity will not be greater than the average horizontal component across a section of the billet. Thus

$$(V_t)_{max} = (V_x)_{max}/\cos\alpha \leq V_1t_1/(t_2\cos\alpha) \qquad (9)$$

so that we obtain the lower bound

$$F \geq F° - \int_{S_2}^{S_1} T°\, ds/(t_2\cos\alpha) \qquad (10)$$

For an upper bound we start with any incompressible velocity field V_i* which satisfies (3), define a strain field by $\dot{\varepsilon}_{ij}^* = (1/2)(V_{i,j}^* + V_{j,i}^*)$ and a stress field σ_{ij}^* by the normality flow rule, and define the dissipation

$$D^* = \frac{1}{2}\int \sigma_{ij}^*\dot{\varepsilon}_{ij}^* \qquad (11)$$

The principle of virtual work shows that

$$\frac{1}{2}\int \sigma_{ij}\dot{\varepsilon}_{ij}^* = t_1V_1F - \int_{S_2}^{S_1} TV_t^*ds \qquad (12)$$

Subtracting both sides of (12) from D* and using Drucker's postulate [1] we obtain

$$D^* - t_1V_1F + \int_{S_2}^{S_1} TV_t^*ds = \frac{1}{2}\int(\sigma_{ij}^*-\sigma_{ij})\dot{\varepsilon}_{ij}^* \geq 0 \qquad (13)$$

whence

$$t_1V_1F - (V_t{}^*)_{max} \int_{S_2}^{S_1} Tds \leq D^* \qquad (14)$$

Finally, we use (5) to obtain the upper bound

$$F \leq \frac{D^*}{t_1V_1}\left[1 - \frac{(V_t{}^*)_{max}/V_1}{\cos\alpha + \sin\alpha/\mu}\right]^{-1} \qquad (15)$$

Alternative bounds can be obtained by considering the extreme cases of perfectly smooth and perfectly rough dies. Obviously the yield-point load for the coulomb die will lie between these two extremes. Therefore, any upper bound on the rough die or any lower bound on the smooth one will also be a corresponding bound on the coulomb die.

A perfectly rough die can permit motion only when T = k. Starting with any kinematically admissible velocity field $V_i{}^*$, we proceed as above but replace T by k in (12), thus leading to the alternative upper bound

$$F \leq (1/t_1V_1)[D^* - k \int_{S_2}^{S_1} V_t{}^*ds] \qquad (16)$$

For the prefectly smooth punch, no shearing force can be supported along AB, so that Eq. (3c) is replaced by

$$\text{On AB and A}'\text{B}'\text{:} \quad T = 0 \qquad (17)$$

Denoting a statically admissible field under this changed boundary condition by $\sigma_{ij}^{\circ\circ}$, we obtain the lower bound of conventional limit analysis

$$F \geq F^{\circ\circ} \qquad (18)$$

In closing, we emphasize two restrictions on the results herein. In conventional limit analysis, the minimum upper bound is equal to the maximum lower bound and this common value is the exact yield-point load. Here, however, the maximum

of all (10) or (18) will generally be distinctly less than the minimum of all (15) or (16), so that we can never do better than define a range within which the yield-point load may lie. Secondly, our results depend upon the assumption that V_t is strictly positive and bounded by (9). Although this assumption is physically reasonable, we have not proved its necessity.

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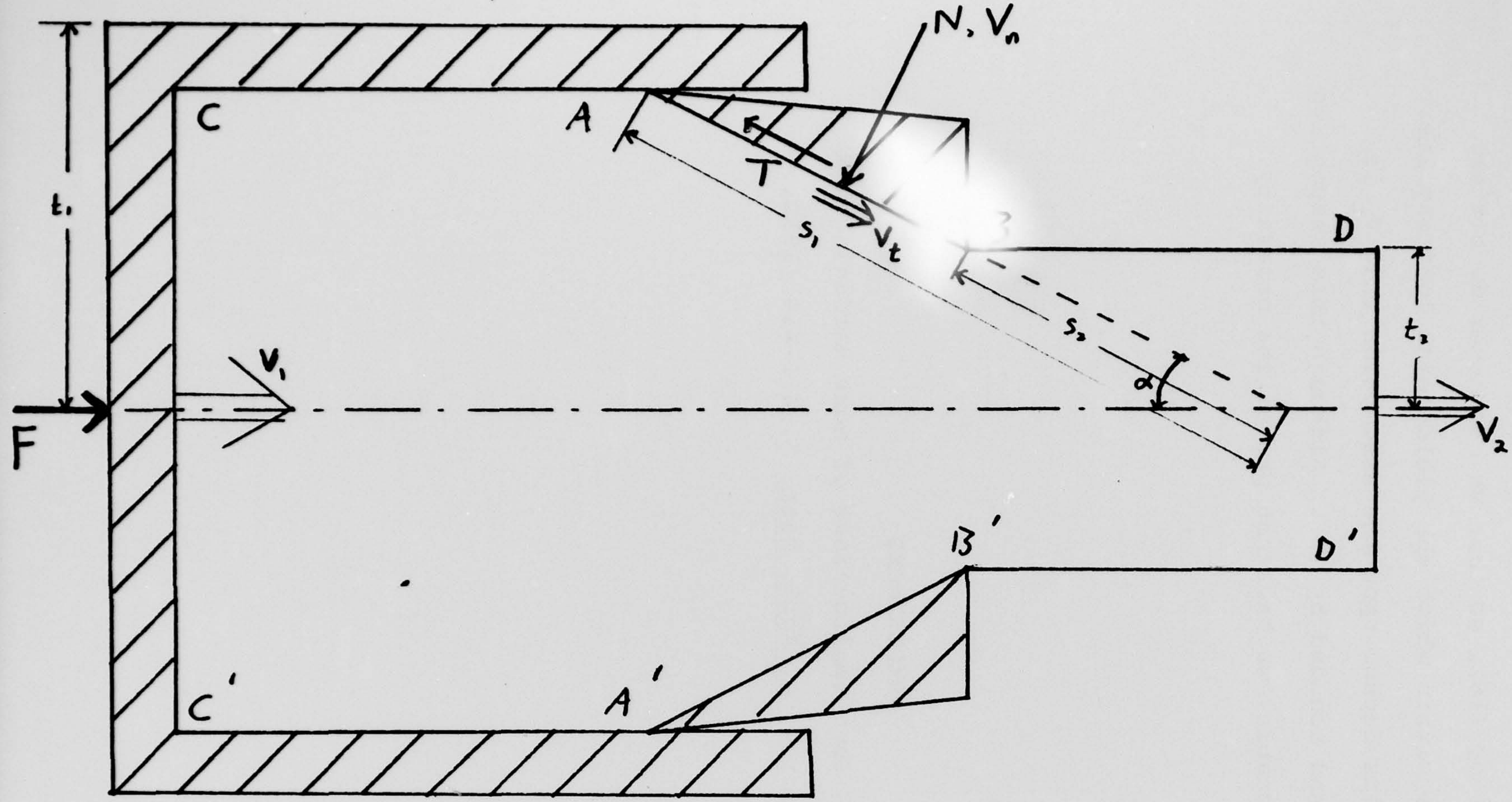

Fig. 1

Inverted Extrusion